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Meson spectrum in a relativistic harmonic model with instanton-induced interaction

K.B. Vijaya Kumar[1], B. Hanumaiah[1], and S. Pepin[2,a]

[1] Department of Physics, Mangalore University, Mangalagangotri, Mangalore 574 199, India
[2] Max-Planck-Institut für Kernphysik, Postfach 103980, D-69029 Heidelberg, Germany

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Abstract. On the basis of the phenomenological relativistic harmonic model for quarks, we have obtained the ground-state masses of the light pseudo-scalar and vector mesons. The full Hamiltonian used in the investigation has Lorentz scalar + vector confinement potential, along with one-gluon-exchange potential (OGEP) and the instanton-induced quark-antiquark interaction. A good agreement is obtained with the experimental masses. The respective role of instanton-induced interaction and OGEP for the determination of the meson masses is discussed.

PACS. 12.39.Ki Relativistic quark model – 12.39.Pn Potential models – 14.40.Aq π, K, and η mesons

1 Introduction

Non-relativistic quark models (NRQM) have proved very successful in describing hadronic properties [1–7]. The Hamiltonian of these quark models usually contains three main ingredients: the kinetic energy, the confinement potential and a hyperfine-interaction term, which has often been taken as an effective one-gluon-exchange potential (OGEP) [8]. Other types of hyperfine interaction have been introduced in the literature; from the non-relativistic reduction of the t'Hooft interaction [9,10], one can derive an Instanton-Induced Interaction (III), which has already been successfully applied in several studies of the hadron spectra [7,11–15]. The main achievement of the III in hadron spectroscopy is the resolution of the $U_A(1)$ problem, which leads to a good prediction of the masses of η and η' mesons. The Goldstone-Boson-Exchange interaction introduced by Glozman and Riska [16] furnishes another example of hyperfine interaction; it allows a good description of the baryon spectrum, and gives in particular a correct ordering for the positive- and negative-parity states. The model of Glozman and Riska has however the major caveat to apply only to baryons and is thus not able to give a unified description of the spectrum of hadrons. There are other models, which, like the III, include an isospin-dependent interaction: the algebraic model [17] or the hypercentral constituent quark model [18], for example.

The successes of the NRQM in describing the spectrum of light hadrons are somehow paradoxical, as light quarks should in principle not obey a non-relativistic dynamics. This paradox has been avoided in many works based on the constituent quark model by using for the kinetic energy term of the Hamiltonian a semi-relativistic or relativistic expression (see, for example, [13,19,20]). In our present work, we have made use of the relativistic harmonic model (RHM) [21]. The RHM combined with OGEP has already been used to calculate light hadrons masses, baryons magnetic moments, leptonic decay widths and N-N scattering phase shifts [22–24]. The full Hamiltonian used in our investigation has Lorentz scalar + vector confinement potential along with OGEP and III. Note that we have neglected the Lorentz structure of OGEP and III. There are terms, in the Fermi-Breit interaction, which are momentum-dependent but they are generally small [25].

In the present work, we have computed the masses of the light pseudoscalar and vector mesons by including the instanton-induced interaction as a short-range non-perturbative gluon effect in addition to the pertubative conventional OGEP derived from QCD. If OGEP is taken as the only source of hyperfine interaction, the value of the strong-coupling constant (α_s) necessary to reproduce the hadrons spectrum is generally much larger than one; this leads to a large spin-orbit interaction, which destroys the overall fit of the spectrum. We think, nevertheless, that it would be exaggerated to eliminate OGEP completely for light quarks. The inclusion of III will diminish the relative importance of OGE for the hyperfine splittings. One

[a] *Present address*: rue de Sluse 13, B-4000 Liège, Belgium; e-mail: stephane_pepin@yahoo.fr

of the aims of this study is to determine explicitly the role played by instantons in meson spectra, when used in the framework of the RHM and to compare the effects of III to the ones of OGE.

The total energy or the mass of the meson is obtained by calculating the energy eigenvalues of the Hamiltonian in the harmonic-oscillator basis spanned over a space extending up to the radial quantum number $n_{\rm MAX} = 5$. The masses of the ground-state mesons are obtained after diagonalization for various values of $n_{\rm MAX}$.

In the next section, we review briefly the relativistic harmonic model and describe the OGE and III terms of our Hamiltonian. We also discuss the parameters involved in our model. The results of the calculations are presented in sect. 3. Some conclusions are given in sect. 4.

2 The relativistic harmonic model

In the RHM, quarks in a hadron are confined through the action of a Lorentz scalar plus a vector harmonic-oscillator potential

$$V_{\rm conf}(r) = \frac{1}{2}(1+\gamma_0)\ A^2 r^2 + M\,, \tag{1}$$

where γ_0 is the Dirac matrix:

$$\gamma_0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\,, \tag{2}$$

M is the quark mass parameter and A^2 the confinement strength. They have a different value for each quark flavour. In the RHM, the confined single quark wave function (Ψ) is given by

$$\Psi = N\ \begin{pmatrix} \Phi \\ \frac{\boldsymbol{\sigma}\cdot\boldsymbol{P}}{E+M}\ \Phi \end{pmatrix} \tag{3}$$

with the normalization

$$N = \sqrt{\frac{2(E+M)}{3E+M}}\,; \tag{4}$$

E is an eigenvalue of the single-particle Dirac equation with the interaction potential given by (1). We perform a similarity transformation to eliminate the lower component of Ψ such that

$$U\ \Psi = \Phi\,, \tag{5}$$

where U is given by

$$\frac{1}{N\ \left[1+\frac{P^2}{(E+M)^2}\right]} \begin{pmatrix} 1 & \frac{\boldsymbol{\sigma}\cdot\boldsymbol{P}}{E+M} \\ -\frac{\boldsymbol{\sigma}\cdot\boldsymbol{P}}{E+M} & 1 \end{pmatrix}\,. \tag{6}$$

Here U is a momentum and state (E) dependent transformation operator. With this transformation, the upper component Φ satisfies the equation

$$\left[\frac{P^2}{E+M} + A^2 r^2\right]\ \Phi = (E-M)\ \Phi\,, \tag{7}$$

which is like the three-dimensional harmonic-oscillator equation with an energy-dependent parameter Ω_n^2:

$$\Omega_n^2 = A\ (E_n + M)^{\frac{1}{2}}\,. \tag{8}$$

The eigenvalue of (7) is thus given by

$$E_n^2 = M^2 + (2n+1)\ \Omega_n^2\,. \tag{9}$$

Note that eq. (7) can also be derived by eliminating the lower component of the wave function, using a Foldy-Wouthuysen transformation, as has been done in [21].

The total energy of the hadron is obtained by adding the individual contributions of the quarks. The spurious centre of mass (CM) is corrected [26] by using intrinsic operators for the $\sum_i r_i^2$ and $\sum_i \nabla_i^2$ terms appearing in the Hamiltonian. This amounts to just subtracting the CM motion zero contribution from the E^2 expression. It should be noted that this method is exact for the $0S$ state quarks as the CM motion is also in the $0S$ state.

We come now to the description of the quark-antiquark potential; it is given by the sum of a one-gluon-exchange and of an instanton-induced interaction potential:

$$V_q(r_{ij}) = V_{\rm OGEP}(r_{ij}) + V_{\rm III}(r_{ij}), \tag{10}$$

with r_{ij} the inter-quark distance.

Among the several versions of the one-gluon-exchange potential $V_{\rm OGEP}$, we have used the following one, first derived in [8] from the QCD Lagrangian in the non-relativistic limit and used subsequently by many authors (for example, [27,28]):

$$V_{\rm OGEP}(r_{ij}) = \frac{\alpha_{\rm s}}{4}\ \boldsymbol{\lambda}_i\cdot\boldsymbol{\lambda}_j \times\left[\frac{1}{r_{ij}} - \frac{\pi}{M_i M_j}\ (1+\frac{2}{3}\ \boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j)\ \delta(r_{ij})\right]\,, \tag{11}$$

where the first term is the residual Coulomb energy and the second term the chromo-magnetic interaction leading to the hyperfine splittings.

To model the instanton-induced interaction, we have used the form given in [7,13]:

$$V_{\rm III}(r_{ij}) = \begin{cases} -8\ g\ \delta(r_{ij})\ \delta_{S,0}\ \delta_{L,0}\,, & \text{for } I=1\,, \\ -8\ g'\ \delta(r_{ij})\ \delta_{S,0}\ \delta_{L,0}\,, & \text{for } I=1/2\,, \\ 8\begin{pmatrix} g & \sqrt{2}g' \\ \sqrt{2}g' & 0 \end{pmatrix}\delta(r_{ij})\delta_{S,0}\delta_{L,0}\,, & \text{for } I=0\,. \end{cases} \tag{12}$$

In the above expression, S, L, I are, respectively, the spin, the relative orbital angular momentum and the isospin of the system. g and g' are dimensioned coupling constants. As in refs. [7,13], the Dirac delta-function appearing in (12) is regularized and replaced by a Gaussian-like function:

$$\delta(r_{ij}) \to \frac{1}{(\Lambda\sqrt{\pi})^3}\exp\left[-\frac{r_{ij}^2}{\Lambda^2}\right]\,. \tag{13}$$

The parameters of the RHM are the masses of the quarks, $M_u = M_d$ and M_s, the respective confinement

Table 1. Values of the parameters used in our model.

b	0.77 fm
$M_{u,d}$	160.6 MeV
M_s	402.5 MeV
$A^2_{u/d}$	3693.0 MeV fm^{-2}
A^2_s	2934.9 MeV fm^{-2}
α_s	0.2
Λ	0.35 fm
g	0.1348×10^{-4} MeV^{-2}
g'	0.0954×10^{-4} MeV^{-2}

strengths, $A^2_u = A^2_d$, A^2_s, and the oscillator size parameter $b\,(= 1/\Omega)$. They are chosen to reproduce various nucleon's properties: the root mean-square charge radius, the magnetic moment and the ratio of the axial coupling to the vector coupling. The confinement strength $A_{u,d}$ is fixed by the stability condition for the nucleon mass against the variation of the size parameter b,

$$\frac{\partial}{\partial b}\langle N\,|H|\,N\rangle = 0. \tag{14}$$

The parameters associated to the strange quark M_s and A^2_s have been fitted in order to reproduce the magnetic moments of the strange baryons, according to the procedure described in [29].

We need also to fit the four parameters of the quark-antiquark interaction. The parameters of III, g, g' and Λ, the strength and the range of the interaction of III, were fitted to the experimental masses of π and K mesons. The coupling constant α_s of OGEP is fitted on the mass of the ρ meson, as III does not contribute to vector mesons. The values of the parameters used in our calculations are listed in table 1. One can notice that our value of α_s (0.2) is much smaller than one.

Table 2. Pseudo-scalar meson masses (in MeV) for successive values of n_{MAX}.

n_{MAX}	π	K	η	η'
0	433.22	685.26	663.70	1388.52
1	280.56	583.34	600.21	1009.92
2	156.78	503.48	555.45	985.87
3	142.25	494.37	550.71	967.59
4	139.65	492.77	549.95	964.41
Expt.	138.04	495.01	547.3	957.78

Table 3. Vector meson masses (in MeV) for successive values of n_{MAX}.

n_{MAX}	ρ	ω	K^*	ϕ
0	771.98	779.84	891.82	1023.06
1	771.77	778.92	891.02	1021.05
2	771.74	778.39	890.75	1020.41
3	771.73	778.36	890.72	1020.37
4	771.72	778.19	890.72	1020.36
Expt.	769.3	783	893.14	1019.417

Table 4. Diagonal contributions of the III and of the chromo-electric and chromo-magnetic part of OGE to the $0S$ state.

Meson	III (MeV)	Colour-electric OGEP (MeV)	Colour-magnetic OGEP (MeV)
π	−245.97	−42.41	−69.43
K	−174.07	−63.27	−27.70
η	−183.96	23.83	−23.16
η'	658.30	−90.30	−103.88
ρ		−42.41	23.14
ω		−39.84	26.83
ϕ		−71.58	13.69
K^*		−63.27	9.23

3 Results and discussion

In our present study of meson spectroscopy, the product of the quark-antiquark oscillator wave functions are expressed in terms of oscillator wave functions corresponding to the relative and CM coordinates using the Moshinsky transformations [30]. We have restricted the CM wave functions to the $0S$ state. The total energy or the mass of the meson is obtained by calculating the energy eigenvalue of the Hamiltonian in the harmonic-oscillator basis spanned over a space extending up to the radial quantum number $n_{\mathrm{MAX}} = 5$.

The masses of the pseudo-scalar and vector mesons after diagonalization for successive values of n_{MAX} are listed in table 2 and table 3, respectively. Table 4 shows the respective diagonal contributions of the III and of the chromo-electric and chromo-magnetic part of OGE to the $0S$ state of each of the mesons.

We get a very good agreement with the experimental masses [31] of the ground-state pseudo-scalar and vector mesons. Both OGEP and III are attractive inside π, the diagonalization in the space of radially excited pion states brings down the value of π to the physical mass. For example, with $n_{\mathrm{MAX}} = 0$, the naive masses of the π and K mesons turned out to be 433 and 685 MeV, respectively. For vector mesons, one can see from table 3 that there is no change in the masses for increasing n_{MAX} as OGEP is repulsive and hence perturbative techniques are fully adequate and justified. We have also calculated that the colour-electric term of OGEP contributes significantly to the masses.

Without III, the naive mass of the pion turns out to be 712 MeV. The contribution of III is thus essential for reducing the masses of pseudo-scalar mesons. If one uses only OGEP, one needs to choose $\alpha_s = 1.9$ in order to reproduce the physical mass of the pion. But this value of α_s leads to much too large masses for the vector mesons.

4 Conclusions

In this work, we have investigated the effect of the Instanton-Induced Interaction on the ground-state masses of light mesons in the framework of the Relativistic Harmonic Model. We have shown that the computation of mesonic masses/mass splittings using OGEP only is inadequate for pseudo-scalar and vector mesons. There is a substantial attractive contribution from III to pseudo-scalar mesons. The same force is responsible for the difference in the mass between η-η' mesons. A consistent description of the π-ρ splitting, the η-η' mixing and the K-K^* splitting is obtained. To obtain the vector meson masses, OGEP is sufficient. Hence, it is justified to use a combination of OGE (with a relatively small strength) and III potentials for pseudo-scalar mesons. In our work, CM corrections have been included exactly. For attractive OGEP and III within pseudo-scalar mesons, the contribution from the off-diagonal elements is found to be significant. The diagonalization of the interaction matrix in the RHM states leads to a lowering of the masses for the pseudo-scalar particles, so as to agree with the experimental masses. Of course, this work is only exploratory as only the ground-state mesons have been studied. The goal was to combine the advantages of the relativistic approach of the RHM with the instanton-induced interaction. This work could be extended by including some tensor and spin-orbit forces in our model. Moreover, it may be also a starting point for further investigation in baryon spectroscopy. Work in this direction is in progress.

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